Signal model for dynamic spectrum allocation close to the cell border of a primary transmitter

SHORT PAPER

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***Abstract*—In this paper we investigate the problem of decision making in dynamic spectrum allocation radios by using energy detection while being close to the coverage area of a primary transmitter. Currently, the signal model used for dynamic frequency selection assumes that the primary signal is either absent or present with a fixed level. However, this model fails to capture the power distribution close to the cell border and thus the prediction capability of the related analytical approaches is rather limited. In this paper we calculate the distribution of the signal power outside the primary cell and then we approximate it by using uniform distribution. On one hand, the proposed approximation allows to estimate the decision performance in a closed-form. On the other hand, the simulations illustrate that the proposed model has better agreement with realistic system behavior compared to the conventional model. The proposed model allows to design the detector such that the system performance in terms of spectrum reuse is improved.**

I. Introduction

In this paper we analyze the primary user signal detection. Particularly, we scrutinize the detection near the primary user coverage area. For instance, such situation occurs if one attempts to detect TV broadcast signal outside but close to the TV coverage area. This type of detection is prerequisite for dynamic spectrum allocation (DSA) by secondary users. The signal identification can be framed as energy detection problem which can be treated as a hypothesis test: The usual detection algorithms assume that the signal either exists with some constant level, hypothesis H_1 or it is zero, hypothesis H_0. We refer this detection scheme as on-off model.

The on-off model describes erroneously the signal power outside the primary cell. Indeed, outside the TV broadcast service area, cell, the signal power is not zero. According to [2] the SNR requirement for TV stations is approximatelly 34dB. However the power of the TV signal reaches the noise level well beyond the cell border. Therefore it is wrong to assume that under hypothesis H_0 only the noise is present. Another simplified assumption adopted by the simplest energy detector design assignes equal prior probabilities for the existence or not of the primary signal [3] [4].

These two simplified assumptions result in mismatch between the simulations and the analytical prediction of the model. In DSA design the on-off model estimates high spectrum reuse probability that does not occur in practice. As a result, the model cannot be used to design reliable detection algorithms. For distributed schemes, the discrepancy between system behavior and analytical prediction is intensified.

In this paper we assign the a priori probabilities based on a geometrical approach and thus we link the decision performance to the system area. Regarding the distribution of the signal power outside the primary cell, we claim that the uniform distribution is a good approximation. This approximation allows to account for primary signal with low level in the presence of hypothesis H_0.

We demonstrate the utility of our model for single and multi-user cooperative energy detection. For distributed energy detection we look into centralized and decentralized systems with fusion. The proposed model allows to evaluate the decision performance in a closed-form for both systems.

II. System Environment Modelling

We consider a square area S where secondary users opt for DSA. The primary system consists of a TV broadcaster that is located at the centre of S and TV receivers. We consider S to be large enough such that the SNR at its border is well below the SNR requirement of TV receivers. The secondary users form a short range ad hoc network. They are allowed to reuse the frequency only if the primary signal level, P_S, at reuse location is lower than P_ϵ. P_ϵ is a system design parameter used to discriminate between high signal level, hypothesis H_1 and low signal level, hypothesis H_0. We set P_ϵ close to the noise level in order to establish a guard area between the TV receivers and the reuse location. The users measures the channel and identify the primary signal strength with respect to P_ϵ. We consider the channel activity due to a single primary transmitter.

Unlike the common practice we link the prior probabilities to the area geometry and the signal level P_ϵ. For uniform distribution of users, the prior probabilities equal the ratio of the area corresponding to each hypothesis divided by S.

$$H_1 : Pr_1 = \frac{\pi \cdot R^2}{S}, \quad H_0 : Pr_0 = \frac{S - \pi \cdot R^2}{S} \tag{1}$$

where R is related to P_ϵ through the channel model.

In the absence of slow fading the shape of the area where $P_S = P_\epsilon$ is circular (Fig. 1). We assume the shape of S being square because its repetition can cover a large area without leaving any open spaces. The application of our model to hexagonal-shaped areas is straightforard. However,

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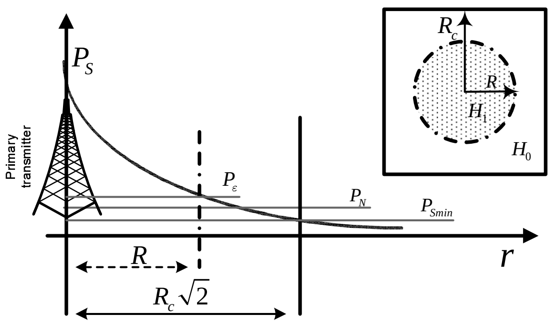

Fig. 1. Decision areas and critical signal levels.

the calculation of the prior probabilities in (1) needs to be modified.

The secondary user measures the channel activity by utilizing a commonly accepted model for energy detection [1]. The $K/2$ complex samples contains noise and signal that are both modelled as Gaussian random variables with zero mean. The user squares the samples and compares their sum, L, with the threshold η. The variable L is a sufficient statistic and follows Chi-square distribution [5]. According to the on-off model there is no signal under H_0. The variance of the samples is the noise power $P_N = N_0 \cdot W$, where N_0 stands for the one-sided power spectrum density of noise and W for the positive signal bandwidth. One can observe in Fig. 1 that the signal level P_S under hypothesis H_0 takes values between P_{Smin} and P_ϵ. P_{Smin} stands for the signal power at the corners of S. Since the signal level ranges over a set of values, we propose to express the distribution of L under H_0 as a composite hypothesis:

$$p\left(L|H_0\right) = \int\limits_{P_{Smin}}^{P_\epsilon} p\left(L|P_S\right) \cdot p\left(P_S|H_0\right) dP_S \qquad (2)$$

where $p(L|P_S)$ is the PDF for fixed signal power and $p(P_S|H_0)$ stands for the PDF of the signal power under H_0. In order to derive $p(L|P_S)$, we model every sample as Gaussian random variable with variance $(P_N + P_S)$. Therefore the distribution of the decision variable L is Chi-square:

$$p\left(L|P_S\right) = \frac{L^{K/2-1} \cdot \exp\left(-L/2\left(P_N + P_S\right)\right)}{2^{K/2} \cdot \left(P_N + P_S\right)^{K/2} \cdot \Gamma\left(K/2\right)} u\left(L\right) \qquad (3)$$

where $\Gamma(\cdot)$ is the upper incomplete Gamma and $u(\cdot)$ is the unit step function. We calculate $p(P_S|H_0)$ and substitute it into (2). To the best of our knowledge the resulting equation does not have a closed-form solution. Therefore in section III we propose to approximate $p(P_S|H_0)$ with uniform distribution.

III. Performance Metrics

In order to evaluate the quality of our approximation we calculate the interference increase and the spectrum reuse probabilities. These metrics allow to quantify the difference between the proposed, the on-off model and the system simulations. The interference is increased when primary signal with high level is missed. The spectrum is reused when primary signal with low level is identified.

The conversion of the distance to the power is due to the channel model, $P_S = P_{Tx} \cdot r^{-n}$ where r is the distance, n the pathloss exponent and P_{Tx} is the power of the primary transmitter. For uniform distribution of users, the CDF of the distances in the area of H_0 can be obtained by evaluating the size of the area under H_0 and divide it by S. The PDF of the distances is derived by differentiating CDF and equals:

$$p\left(r\right) = \begin{cases} \frac{2\cdot\pi\cdot r}{S}, & R < r < R_c \\ \frac{4\cdot r}{S} \cdot \left(2\sin^{-1}\left(\frac{R}{r}\right) - \frac{\pi}{2}\right), & R_c < r < \sqrt{2} \cdot R_c \end{cases} \qquad (4)$$

where R_c is the radius of the inscribed circle to S (Fig. 1).

Since the random variable P_S is continuous function of r, the distribution $p(P_S|H_0)$ can be determined. Unfortunately, by substituting it into (2) we are not able to acquire a closed-form expression. We propose to approximate the signal distribution under H_0 with uniform distribution. After using $p(P_S|H_0) \approx 1/(P_\varepsilon - P_{Smin})$ in (2) we are able to express $p(L|H_0)$ in closed form. The reuse probability equals the left tail integral of $p(L|H_0)$ weighted with the a priori probability Pr_0. By using the series expansion of $\gamma(\cdot)$ we express the reuse probability in (5). Therein, $\gamma(\cdot)$ stands for the lower incomplete Gamma function. On the other hand, for the on-off model the distribution $p(L|H_0)$ is derived from (3) for $P_S = 0$ and the spectrum reuse is:

$$Pr_{reuse} = Pr_0 \cdot \frac{1}{\Gamma\left(K/2\right)} \cdot \gamma\left(\frac{K}{2}, \frac{\eta}{2 \cdot P_N}\right) \qquad (6)$$

In the area of H_1 the signal power is set to be P_ϵ. This worst case condition, adopted both by the on-off and the proposed model, is a method to protect the primary system against interference increase [3] [4]. The distribution $p(L|H_1)$ is derived from (3) by setting $P_S = P_\epsilon$. The miss probability equals the left tail integral of $p(L|H_1)$ weighted with the a priori probability Pr_1.

In order to determine the decision threshold at the user, the system designer should constraint the miss probability according to the primary system outage probability: $Pr_{miss} \leq Pr_{outage}$. Obviously, there are multiple feasible solutions to this problem. The common practice is to select the one that maximizes the total number of correct decisions. The problem of identifying the optimal decision level can be expressed as:

$$\begin{array}{ll} Maximize: & Pr_{reuse}\left(\eta\right) + Pr_1 - Pr_{miss}\left(\eta\right) \\ Subject\ to: & Pr_{miss}\left(\eta\right) \leq Pr_{outage} \end{array} \qquad (7)$$

The optimization algorithm works as following. For increasing decision threshold η we calculate the objective function $Pr_{reuse}\left(\eta\right) + Pr_1 - Pr_{miss}\left(\eta\right)$ until the design constraint is not satisfied. From all feasible $\eta' s$ we select the one that maximizes the objective function. It is a matter of future work to show that the identified solution is globally optimal. Next, we explain the reasons for selecting the uniform distribution to approximate the signal power outside the cell border.

$$Pr_{reuse} = \frac{Pr_0}{P_\epsilon \cdot (K/2-1)} \cdot \sum_{j=1}^{K/2-1} \frac{(P_N + P_\epsilon)\cdot\gamma\left(j, \frac{\eta}{2\cdot(P_N+P_\epsilon)}\right) - (P_N + P_{Smin})\cdot\gamma\left(j, \frac{\eta}{2\cdot(P_N+P_{Smin})}\right)}{(j-1)!} \quad (5)$$

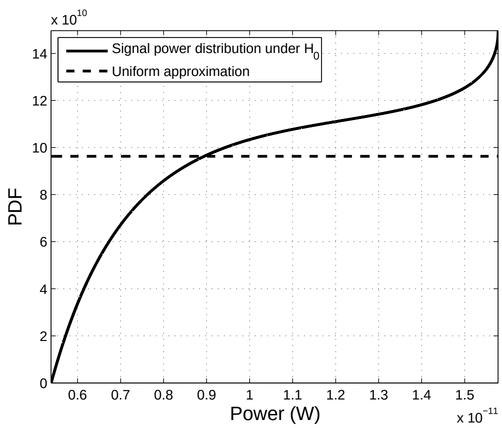

Fig. 2. Signal power distribution under H_0 for $R = R_C$.

IV. Uniform signal power approximation

In our opinion, the limited prediction capability of the on-off model occurs because the model does not consider the existence of primary signal power under H_0. On the other hand, the proposed model takes into account the presence of primary signal with low power and thus it represents the system environment more accurately. It was shown in section III that the uniform approximation empowers the designer with analytical expression for evaluating the reuse probability. The exponential or the power law reflects better the signal power distribution but one should resort to numerical methods to estimate the spectrum reuse.

In order to justify our selection we look more closely to what kind of system the uniform power distribution corresponds to; how it is related to the user distribution. In Fig. 2 we plot the distribution of the signal power under H_0 for uniform distribution of users. Compared to that system the uniform power approximation implies that more users experience low mean signal level. Such low power is observed close to the outer border of S. Correspondingly, one can interpret the uniform power distribution describing a system where user concentration near the outer border of S is somewhat higher than in the rest of S.

V. Distributed Scheme

The single user detection suffers from poor performance. A common approach to overcome this problem introduces many secondary users with fusion [4] [6]. The improved characteristics accompanying our model allows to express analytically the performance and estimate the distributed system behavior more precisely compared to the on-off model.

The problem of combining multiple user power measurements resembles distributed decision with fusion [7] [8] [9]. In this context a cluster of users senses the spectrum and jointly decides whether to reuse a particular frequency or not. The distributed system performance is impacted by the amount of information conveyed from the users to the fusion [8] [9]. The two extremes here are the centralized scheme where the exact measurement is conveyed and the decentralized scheme where a hard decision is communicated. The performance of the two schemes is affected by different parameters. The performance of the centralized scheme depends on the decision threshold set at the fusion. The performance of the decentralized scheme depends both on the quantization levels set at the users and on the decision rule employed by the fusion centre [8] [9].

Our model and the on-off model describe the system in different manner. Because of that the optimized rules are in general different. We show that our model establishes better decision rule that results in higher spectrum reuse and matches better with simulation results.

A. Centralized scheme

Let assume there are N_S secondary users deployed inside a cluster that belongs to S. Each user collects $K/2$ complex samples and sends them to the fusion. The secondary user cluster size is relatively small compared to S. Because of that we assume that all the users experience the same primary signal level mean. That is, the user measurements are essentially drawn from the same distribution, with the same mean and variance. In addition, we assume that the user measurements are statistically independent. Under these assumptions, the fusion processes $N_S \cdot K/2$ independent samples and decides the frequency availability on behalf of the users. Since the samples are not preprocessed at the users the decision problem is the same as the problem of a single user with more samples. The optimal fusion rule squares the samples and compares their sum with a threhsold. The optimal threshold is calculated by (7), separately for the on-off and the proposed model.

B. Decentralized scheme

In the decentralized system the users convey hard decisions to the fusion indicating the presence or not of primary signal. The fusion receives the binary decisions and communicates its own decision back to the users. The simplest rule employed by the users compares the sum of the received samples with the local threshold η. If the sum is larger than η, $+1$ is conveyed to the fusion; otherwise, -1 is communicated. Given η, the optimal fusion rule compares the weighted sum of the individual decisions with another threshold [7]. Provided that all the users have identical η, the optimal fusion rule becomes the K_S out of N_S rule [8]. Although the assumption for identical levels leads to only near optimal solution, it is employed in practice due to the significant reduction of computations. Finally, the

$$Pr^F_{miss} = Pr_1 \cdot \sum_{i=0}^{K_S} \begin{pmatrix} N_S \\ i \end{pmatrix} \cdot \left(1 - \int_0^{\eta} p\left(L|H_1\right) dL\right)^i \cdot \left(\int_0^{\eta} p\left(L|H_1\right) dL\right)^{N_S - i} \tag{8}$$

$$Pr^F_{reuse} = Pr_0 \cdot \sum_{i=0}^{K_S} \begin{pmatrix} N_S \\ i \end{pmatrix} \cdot \left(1 - \int_0^{\eta} p\left(L|H_0\right) dL\right)^i \cdot \left(\int_0^{\eta} p\left(L|H_0\right) dL\right)^{N_S - i} \tag{9}$$

fusion decides not to reuse the frequency provided that more than K_S users vote for hypothesis H_1. One can foresee that the optimal test for the decentralized system needs the optimal pair (η, K_S) to be derived. Before doing that we compute the performance in terms of K_S and η:

We assume that the user measurements are independent and no communication channels exist between the users. Under these assumptions the hard decisions of the users are statistically independent. Therefore the test statistic at the fusion centre follows binomial distribution. Under hypothesis H_1 the binomial parameters are N_S and $(1 - \int_0^{\eta} p\left(L|H_1\right) dL)$, the probability of detection at the users. The miss probability at the fusion occurs when only up to K_S users decide correctly. It is calculated in (8). Similarly, under hypothesis H_0, the test statistic at the fusion follows binomial distribution with parameters N_S and $(1 - \int_0^{\eta} p\left(L|H_0\right) dL)$. The reuse probability is calculated in (9).

In order to derive the optimal test for the decentralized system we follow the same practice as for the single user system. The optimization problem is formulated as:

$$\begin{array}{ll} Maximize: & Pr^F_{reuse}\left(K_S, \eta\right) + Pr_1 - Pr^F_{miss}\left(K_S, \eta\right) \\ Subject\ to: & \qquad Pr^F_{miss}\left(K_S, \eta\right) \leq Pr_{outage} \end{array} \tag{10}$$

The optimal solution is obtained using the following algorithm: We fix K_S and derive η that satisfies the design constraint and maximizes the total number of correct decisions. In order to construct the design constraint we evaluate (3) for $P_S = P_\epsilon$, obtain $p(L|H_1)$ and substitute it into (8). In order to obtain Pr^F_{reuse} we evaluate $p(L|H_0)$ both for the on-off and the proposed model and substitute them into (9). For fixed K_S, we increase η until the design constraint is not satisfied. After repeating the procedure for all K_S we end up with N_S feasible pairs (K_S, η). The optimal pair is the one maximizing the total number of correct decisions.

VI. Numerical results

We illustrate the benefits of our model by comparing its performance to the on-off model. Both models are validated with simulations obtained from a realistic system set up with the following parameters: At the centre of the area S the primary transmitter uses $P_{Tx} = 100$ $[kW]$ and the pathloss exponent is $n = 3.1$. The one-sided power spectrum density of noise equals $N_0 = 5 \cdot 10^{-16}$ $[W/Hz]$ and the channel bandwidth is $W = 25$ $[kHz]$.

Recall that P_ϵ is a system design parameter used to separate between low and high signal levels. The sensitivity of the proposed model should be tested for various $P_\epsilon's$. We do it by looking at two different types of variation of parameters: In the first we keep the size of the area S fixed ($R_c \approx 125$ $[km]$) and we change the size of the circular area corresponding to the signal level P_ϵ, $P_\epsilon = P_{Tx} \cdot R^{-n}$. The R is selected such that P_ϵ is increased from 1 $[dB]$ to 5 $[dB]$ above the noise level. In the second we increase S and we keep the cell size fixed such that the P_ϵ is fixed to 1 $[dB]$ above the noise level ($R \approx 125$ $[km]$). Let call the described variations as *test1* and *test2* respectively.

The parameter describing the quality of the spectrum sensing is the maximum reuse that is calculated based on the geometry of the area and by using (1). For both tests it varies approximately from $Pr_0 = 0.21$ to $Pr_0 = 0.56$.

Recall that the centralized system with specifications described in section V is equivalent to a single user system with more collected samples. Therefore it is sufficient to compare the two models for the two distributed schemes only. Particularly, we consider distributed system of $N_S = 5$ users. The primary system outage probability equals $Pr_{outage} = 0.01$. The miss probability at the fusion must satisfy $Pr^F_{miss} \leq 0.01$.

Regarding the centralized scheme, the optimal level at the fusion centre is obtained by solving (7) for $N_S \cdot K/2$ samples. For the on-off model we substitute (6) into (7) and for the proposed approximation we substitute (5) into (7). The only information available at the system designer is the signal level P_ϵ. The signal level P_{Smin} is in general unknown. Therefore we set in (5), $P_{Smin} = 0$. This setting indicates the condition when the P_{Smin} is not available and allows to make more fair comparison between the two models. For the decentralized scheme the quantization levels at the users and the optimal fusion rule are obtained by solving (10).

The optimal thresholds derived based on the two models are in general different because the two models evaluate the reuse probability by using different expressions. After acquiring them we create the simulation scenarios. We generate a cluster of 5 users that collect $K/2 = 5$ complex samples each. The size of the cluster is negligible compared to S. For instance, $R_{cluster} = 50$ $[m]$. We obtain the simulated reuse by counting the number of users that belong to H_0 and vote for H_0.

For the *test1* we plot the reuse probability for the centralized and the decentralized system in Fig. 3 and Fig. 4 respectively. The spectrum reuse obtained by simulations is also depicted. In this case, the two models result in different decision rules.

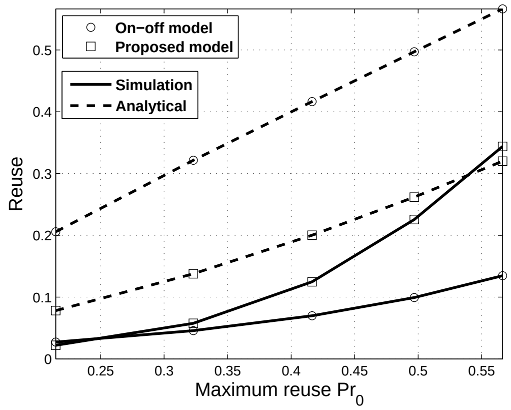

Fig. 3. Reuse for centralized scheme with fixed R_c and variable R.

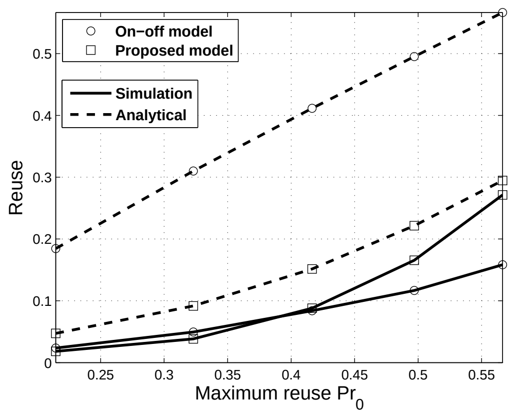

Fig. 4. Reuse for decentralized scheme with fixed R_c and variable R.

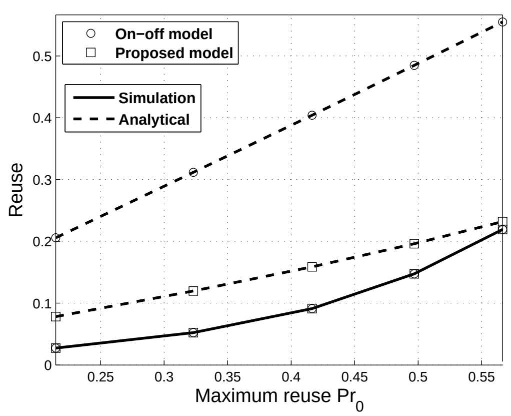

Fig. 5. Reuse for centralized scheme with fixed R and variable R_c.

One can see that the proposed approximation not only fits better to the simulation results but it also has decision level that results in higher reuse. In Fig. 5 and Fig. 6 we compare the two models for *test2*. In this configuration, the two models result in the same decision level. However, the proposed model predicts the behavior of the real system more accurately.

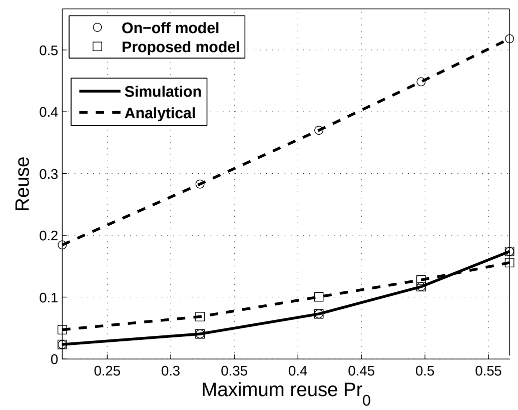

Fig. 6. Reuse for decentralized scheme with fixed R and variable R_c.

VII. Conclusions

The purpose of this paper is to improve the simple energy detection for dynamic spectrum allocation. We augment the simple detector by modeling the primary user power distribution outside of its service area. Also, we incorporated the prior probabilities into the detector design. The proposed extensions are selected such that the problem is still analytically tractable. We illustrated the usability of our model by calculating the performance of the detector in distributed setups. It turns out that the proposed extensions allow to derive an analytical performance prediction that has good match with simulation results in realistic environment. In some cases we were even able to derive decision thresholds that provided better performance compared to the thresholds calculated with conventional models. The results of our analysis are promising and indicate that our model can be useful for network optimization and planning.

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